

# Validation of Equations for Motorcycle and Rider Lean on a Curve

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#### Abstract

Several sources report simple equations for calculating the lean angle required for a motorcycle and rider to traverse a curved path at a particular speed. These equations utilize several assumptions that reconstructionists using them should consider. First, they assume that the motorcycle is traveling a steady speed. Second, they assume that the motorcycle and its rider lean to the same lean angle. Finally, they assume that the motorcycle and rider lean. This study reports physical testing that the authors conducted with motorcycles traversing curved paths to examine the net effect of these assumptions on the accuracy of the basic formulas for motorcycle lean angle. We concluded that the basic lean angle formulas consistently underestimate the lean angle of the motorcycle as it traverses a particular curved path. This finding is consistent with the expected effect of the simplifying assumptions employed by these basic formulas.

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### Introduction

Three factors limit the speed at which a motorcyclist can traverse a curve. The first of these is the limit of the available friction between the motorcycle tires and the roadway. The second is a geometric limit that is defined by the lean angle at which components of the motorcycle (a foot peg, for instance) come into contact with the roadway or at which the geometry of the tire prevents additional leaning. The third is the limit imposed by the rider's psychological limits - their willingness to approach either the geometric or friction limits of their motorcycle [Hugemann, 2013].

For a motorcyclist traversing a flat curve (no superelevation), the friction limit will be reached when the lateral acceleration (in gravitational units) of the motorcycle/rider combination is equal to the coefficient of friction between the roadway and the motorcycle tires. Lambourn examined friction coefficients between motorcycle tires and dry asphalt roadways and found peak friction coefficients of 1.2 [Lambourn, 2010]. With this level of friction, a motorcycle traversing a flat, 250-foot radius curve would have a friction limited speed of 67 mph. On a flat, 500-foot radius curve, the same motorcycle would have a friction limited speed of 95 mph. Using a friction coefficient of 0.8, which is consistent with the sliding friction coefficients reported by Lambourn, a motorcycle traversing a flat, 250-foot radius curve would have a friction limited speed of 55 mph. On a flat, 500-foot radius curve, the same motorcycle would have a friction limited speed of 77 mph. Many motorcycles will not have the geometric clearances necessary for the lean angle that these speeds would require.

The maximum lean angle for most motorcycles will fall within the range of 25 to 50 degrees [Bartlett, 2011]. On a flat, 250-foot curve, a motorcycle that can lean 25 degrees will be able to achieve a speed of

42 mph before components begin to contact the roadway. A motorcycle that can lean 50 degrees would be able to achieve the friction-limited speed of 67 mph. Thus, for many motorcycles, the geometric limit will be more restrictive than the friction limit.

In relationship to the geometric limit of a motorcycle, it is worth mentioning that suspension loading and compression that occur when a motorcycle travels around a curve will have an effect on the geometric limit for a motorcycle on a curve. As the load on the suspension increases, the springs will compress more and the ground clearance of components on the motorcycle will decrease. A previous study [Rose, 2014] reported physical testing of a 2003 Harley Davidson FXD motorcycle that documented the motorcycle's ground clearance under various loading conditions. For the motorcycle tested in this previous study, the authors concluded that, while the suspension loading had an effect on the maximum achievable lean angle, the effect was small. In that instance, the authors concluded that the maximum lean angle for a motorcycle could reasonably be estimated from what is reported in manufacturer specifications. This is likely to be true generally, since the maximum lean angle reported in these specifications will typically have been determined according to the procedure described in Society of Automotive Engineers (SAE) Recommended Practice J1168. This recommended practice specifies that the front and rear suspension systems on the motorcycle be compressed to 75% of their maximum travel. The motorcycle is then leaned until a component contacts the test surface and the lean angle is measured. Further research could, of course, be done on this specific issue to determine if there are motorcycles for which the suspension effects would be significant. Many riders will reach a psychological limit on their willingness to continue to lean their motorcycle before they reach the maximum lean angle of their motorcycle [Bartlett, 2011; Hugemann, 2013]. Watanabe & Yoshida found that the

(1)

maximum lean angles utilized by novice riders were typically in the range of 15 to 25 degrees and those used by experienced riders were in the range of 34 to 40 degrees [Watanabe & Yoshida, 1973]. These results imply that the experienced riders in the study by Watanabe & Yoshida used maximum lean angles that would approach the lean angle limits of many motorcycles, whereas novice riders stopped well short of the motorcycle limits. Using the middle values of these ranges, these results further imply that on a flat, 250-foot curve, an experienced rider would be willing to lean far enough to traverse the curve at a speed of 53 mph.

#### Motorcycle Lean on a Curve

The lean angle required for a motorcyclist to traverse a particular curved path will be the angle that brings the overturning moment generated by the tire frictional forces into balance with the opposing moment generated by the tire forces perpendicular to the road surface. The required lean angle increases with increasing speed and decreasing path radius. Fricke [2010] and Cossalter [2006] report that the lean angle of a motorcycle for a particular path and speed can be calculated with the following equation:

$$\theta = \tan^{-1} \left( \frac{v_{mc}^2}{g \times r} \right)$$

In this equation,  $\theta$  is the lean angle of the motorcycle,  $v_{mc}$  is the motorcycle's velocity, g is the gravitational acceleration, and r is the path radius. Equation (1) yields the lean angle relative to gravity or relative to the vertical. For a flat roadway, this will also be the lean angle relative to the road. However, if the motorcycle is traversing a curve with superelevation, the lean angle relative to the roadway will be different than what Equation (1) yields. Thus, in evaluating the geometric limit of a motorcycle on a curve, superelevation is an important factor to consider.

To see how the superelevation can be incorporated into Equation (1), consider Figure 1, which is a diagram of a motorcyclist traversing a leftward curve with a superelevation. As this diagram shows, the superelevation angle has been given the symbol  $\phi$  and, in this case,  $\theta$  designates the lean angle of the motorcycle relative to the road surface. Thus, the lean angle of the motorcycle and rider relative to the vertical direction is  $\phi + \theta$ . In this derivation, counterclockwise rotations are positive, and therefore, the superelevation and lean angles for a leftward curve are positive. Figure 1 also depicts the forces applied to the cornering motorcycle - the combined weight of the motorcycle and rider applied at their effective center of mass (W), the lateral friction force ( $F_{friction}$ ) applied at the tire contact patch.

Newton's second law dictates that, for the scenario depicted in <u>Figure</u> <u>1</u>, the sum of the forces in the lateral direction are equal to the mass multiplied by the lateral acceleration.

$$\sum F_{lat} = m \times a_{lat}$$

By examination of Figure 1, it can be seen that

$$\sum F_{lat} = \frac{F_{normal} \times \sin \emptyset +}{F_{friction} \times \cos \emptyset}$$

Further, for a motorcycle traversing a curved path

$$a_{lat} = \frac{v_{mc}^2}{r}$$

(4)

(5)

Therefore, Equation (2) can be rewritten as follows:

$$F_{normal} \times \sin \phi + F_{friction} \times \cos \phi$$
$$= \frac{W}{g} \times \frac{v_{mc}^2}{r}$$



Figure 1. Forces Applied to a Cornering Motorcycle

In order to remain in equilibrium as it traverses the curve, the resultant of the normal and friction forces must act along the line connecting the tire contact patch to the center of mass of the motorcycle and rider. For this to be the case, the friction and normal forces must be related to the lean angle as defined by the following equation:

(3)

(2)

$$F_{friction} = F_{normal} \times \tan \theta$$

Substituting Equation (6) into Equation (5) yields:

$$F_{normal} \times (\sin \phi + \tan \theta \times \cos \phi) = \frac{W}{g} \times \frac{v_{mc}^2}{r}$$
(7)

As it traverses the curve, the motorcycle depicted in <u>Figure 1</u> is in static equilibrium in the vertical direction. Therefore, the sum of the forces in the vertical direction is equal to zero. This can be written as follows:

$$-W + F_{normal} \times \cos \emptyset$$
$$-F_{friction} \times \sin \emptyset = 0$$
<sup>(8)</sup>

Substituting Equation (6) into Equation (8) and solving for  $F_{normal}$  yields:

$$F_{normal} = \frac{W}{\cos \emptyset - \tan \theta \times \sin \emptyset}$$

Substituting Equation (9) into (7) yields the following equation:

$$\frac{1}{g} \times \frac{v_{mc}^2}{r} = \frac{\sin \emptyset + \tan \theta \times \cos \emptyset}{\cos \emptyset - \tan \theta \times \sin \emptyset}$$
(10)

Roadway superelevation will not typically exceed 6 or 7 degrees [The Green Book, 1990]. Given this, we can employ small-angle assumptions to simplify Equation (10). Specifically:

$$\cos \phi \approx 1$$
 (11)

$$\sin \varphi \approx \tan \varphi$$

Substituting Equations (11) and (12) into (10) yields the following equation:

$$\frac{1}{g} \times \frac{v_{mc}^2}{r} = \frac{\tan \emptyset + \tan \theta}{1 - \tan \emptyset \times \tan \theta}$$
(13)

Using trigonometric identities, it can be shown that

$$\frac{\tan \emptyset + \tan \theta}{1 - \tan \emptyset \times \tan \theta} = \tan(\emptyset + \theta)$$

Therefore:

$$\theta = \tan^{-1} \left( \frac{v_{mc}^2}{g \times r} \right) - \emptyset$$
<sup>(15)</sup>

(14)

Equation (15) shows that the superelevation angle reduces the required lean angle relative to the road surface and that each degree of superelevation reduces the required lean angle by one degree.

#### Assumptions

Equations (1) and (15) utilize several assumptions that reconstructionists applying them should keep in mind. First, they assume the motorcycle is traveling a steady speed (i.e., not accelerating or decelerating) over the distance the radius is measured. Second, they assume that the motorcycle and its rider have the same lean angle. This will often be an accurate assumption, but a rider has the option of leaning either more or less than they lean the motorcycle and reconstructionists should be attentive to situations where this assumption might not apply. As Cossalter has noted [2006]: "The motorcycle [lean] angle on a turn is influenced, in a significant way, by the rider's driving style. By leaning with respect to the vehicle, the rider changes the position of his center of gravity with respect to the motorcycle...if the rider remains immobile with respect to the chassis, the center of gravity of the motorcycle-rider system remains in the motorcycle plane... if the rider leans towards the exterior of the turn, the center of gravity is also moved to the exterior of the turn with respect to the motorcycle. As a result, he needs to incline the motorcycle further so that the tires, being more inclined than necessary, operate under less favorable conditions... If the rider leans his torso towards the interior of the turn and at the same time rotates his leg so as to nearly touch the ground with his knee, he manages to reduce the roll angle of the motorcycle plane."

Finally, Equations (1) and (15) assume that the motorcycle tires have no width, such that the portion of the tires contacting the roadway does not change as the motorcycle and rider lean. In reality, as the motorcycle leans, the portion of the tire contacting the road changes and the contact patch moves in the direction of the lean (Figure 2). This results in the actual lean angle required for a particular curve being higher than that predicted by Equations (1) or (15).

(6)

(9)

(12)



Figure 2. Change in Contact Patch Location with Tire Lean

## Physical Testing - April 28, 2014

On April 28, 2014, the authors conducted physical testing using a 2012 Suzuki DR650SE Enduro motorcycle (Figure 3). Figure 4 is a photograph that shows a cross-sectional view of the rear tire of this motorcycle and how it contacts the ground when the motorcycle is leaning on its kickstand (with no rider). Additional information about this motorcycle and its riders is presented in <u>Appendix A</u>. Testing with this motorcycle was conducted in the parking lot at Fay Myers Motorcycle World in Greenwood Village, Colorado. Figure 5 is an aerial photograph showing this parking lot. Because this lot is often used for teaching the Motorcycle Safety Foundation's (MSF) riding courses, it is pre-marked with various courses.

The testing reported here utilized the large oval track marked with blue paint, visible near the center of <u>Figure 5</u>. This testing utilized two riders - one a novice and one an expert rider, both of whom had completed the MSF Basic Rider Course. The riders were instructed to make turns at different speeds, while trying to keep their speed constant during the turn. The riders also attempted to lean their body to the same degree that the motorcycle leaned during each turn. Each rider rode the motorcycle around the oval course for a period of time, varying their speed and direction of travel (clockwise or

counterclockwise). The path, speed, and lean angle of the motorcycle were continuously documented using a Racelogic VBOX that measured speed, position, and roll angle at 20Hz. The VBOX system utilized two GPS antenna to measure the test vehicle's translational and angular positions and its speed throughout the tests at a frequency of 20 Hz. A metal crossbar was strapped to the rear of the motorcycle and the GPS sensors were magnetically attached to this crossbar (Figures 3 and 6). One sensor was attached at the motorcycle centerline and another near the left extent of the crossbar. The metal crossbar was damped with a piece of Styrofoam placed between the crossbar and the rack on the back of the bike. The VBOX data logger was placed in the left saddlebag of the motorcycle. This testing was also captured with two video cameras recording at a rate of 30 frames per second. At the time of the testing, the temperature was approximately 48° F, the test surface was bare and dry, and it was windy. The Fay Myers parking lot is in a business district with a number of surrounding buildings and trees that blocked the wind to some degree and, in the judgment of both riders, the wind was not of a magnitude that required significant adjustments to their riding.



Figure 3. Suzuki Motorcycle Used for Testing



Figure 4. Contact between the Suzuki's Tire and the Test Surface



Figure 5. Parking Lot Used for Testing



Figure 6. Motorcycle and Rider at Apex of Curve during One Test

## Physical Testing - July 7, 2014

On July 7, 2014, the authors conducted additional physical testing using a 2007 Kawasaki VN900-D motorcycle (<u>Figure 7</u>). <u>Figure 8</u> is a photograph that shows a cross-sectional view of the rear tire of this motorcycle and how it contacts the ground (this time with a rider). Testing with this motorcycle was again conducted in the parking lot at Fay Myers Motorcycle World in Greenwood Village, Colorado.



Figure 7. Kawasaki Motorcycle Used for Testing



Figure 8. Contact between Kawasaki's Rear Tire and Ground

This testing again utilized two riders - one a novice and one an expert rider, though the expert rider in this set of testing was different than in the first. In this instance, the expert rider had been riding motorcycles since adolescence and, as an adult, had received law enforcement motorcycle rider training. He rode as a motorcycle officer for the Fullerton Police Department for one year and also rode a motorcycle for a funeral escort company for one year.

The riders were given the same instructions and again, each rider rode the motorcycle around the oval course for a period of time, varying their speed and path (Figure 9). The path, speed, and lean angle of the motorcycle were again documented with a Racelogic VBOX that measured speed, position, and roll angle at a frequency of 20Hz. This testing was captured with a video camera recording at a rate of 30 frames per second. At the time of the testing, the temperature was approximately 92° F, the test surface was bare and dry, and the air was calm. Additional information about the motorcycle and riders is presented in <u>Appendix A</u>.



Figure 9. Kawasaki Motorcycle and Rider during Testing

#### **Analysis and Results**

The VBOX data from each cornering maneuver was analyzed to determine the motorcycle's path radius, speed, and lean angle relative to gravity (rather than relative to the road surface). For evaluation of <u>Equation (1)</u>, these values were tabulated for the time during each cornering maneuver when the lean angle reached a maximum. The path radius was determined by fitting a curve to the VBOX positional data for each curve. In order to normalize the results from the testing and plot them all on one graph, the path radius and speed were used to calculate a lateral acceleration for each maneuver.

Figure 10 is a graph that depicts the lateral acceleration and resulting lean angle from each cornering maneuver from our testing compared to the lean angle Equation (1) would predict for each lateral acceleration level. Lateral acceleration in g's is plotted on the horizontal axis and the lean angle in degrees is plotted on the vertical axis. This is the lean angle of the motorcycle itself since the instrumentation was attached to the motorcycle. Points plotted in gray are for the Suzuki motorcycle and points plotted in purple are for the Kawasaki motorcycle. Points plotted with a square are for the novice rider and points plotted with a circle are for the expert riders.

Several trends emerge in Figure 10 that are deserving of comment. First, while Equations (1) and (15) prescribe a particular lean angle for a given lateral acceleration, the test data we gathered shows considerable scatter in the actual lean angle for any particular lateral acceleration level. Clearly there are factors other than lateral acceleration that affect how much a rider leans on a given curved path. Second, the actual lean angle is nearly always greater than the lean angle predicted by Equation (1). Thus, if one were to use Equation (1) - or presumably Equation (15) - to calculate the maximum speed at which a motorcycle could traverse a curve before a foot peg, or some other component, began scraping the ground, these equations would yield a speed greater than the actual geometry-limited speed. If one were to use these equations to calculate the lean angle for a particular curve at a particular speed, the actual lean angle would be higher than calculated. This trend is expected based on the assumptions incorporated by Equation (1).

As discussed in the "Assumptions" section, <u>Equation (1)</u> neglects the width of the motorcycle tire, and thus, neglects the fact that the portion of the tire in contact with the roadway changes as the rider leans the motorcycle. Cossalter showed that the additional lean angle required due to the tire width could be calculated using <u>Equations</u> (<u>16</u>) and (<u>17</u>).

$$\theta = \theta_{Equation (1)} + \Delta \theta_{tire width}$$
(16)
$$\left( \frac{t}{2} \times \sin(\theta_{Equation (1)}) \right)$$

$$\Delta \theta_{tire \ width} = \sin^{-1} \left( \frac{\frac{1}{2} \times \sin(\theta_{Equation \ (1)})}{h - \frac{t}{2}} \right)$$
(17)

In these equations, t is the tire width and h is the combined motorcycle and rider center of gravity height.

Figures 11 and 12 are similar to Figure 10 with the exception that Figure 11 only plots the test points for the Suzuki motorcycle and Figure 12 only plots the points for the Kawasaki motorcycle. These figures also plot Equation (16) for each motorcycle with a dashed line. For purposes of calculating the curve for Equation (16), we used the average of the front and rear tire widths of each motorcycle to calculate t. We estimated the center of gravity height of each motorcycle using Equation (18) below, which is from Cossalter (2002). In this equation, *WB* is the wheelbase of the motorcycle. We estimated that each rider's seated center of gravity height would be at their navel. We then weighed each motorcycle and rider using Intercomp EZ-Weigh scales and calculated a combined center of gravity height for the motorcycle and rider using the formula of Equation (19).

$$CG \ Height_{MC} = 0.3705 \times WB$$

$$(18)$$

$$ight_{Combined} = \frac{CG \ Height_{MC} \times W_{MC} + CG \ Height_R \times W_R}{W_{Total}}$$

Figures 11 and 12 demonstrate that incorporating the tire width into the calculation of lean angle reduces the average error in the calculation. However, including this factor did not move the theoretical relationship to the average of the testing data.

CG He



Comparison between the Test Data and Equation (1)

Figure 10. Test Results Compared to Equation (1)

Comparison between the Test Data and Equation (16) Suzuki



Figure 11. Test Results for Suzuki Compared to Equation (16)



Comparison between the Test Data and Equation (16) Kawasaki

Figure 12. Test Results for Kawasaki Compared to Equation (16)

This research did not measure or quantify the rider lean angle during the cornering maneuvers. In theory, it is possible that a discrepancy between the motorcycle and rider lean angle could explain why the motorcycle lean angle was consistently under-predicted by <u>Equations</u> (1) and (16). If this were the case, it would imply that both the novice and expert riders consistently leaned less than what they leaned their motorcycles. Review of the testing video revealed that the novice rider had a tendency to lean less than the motorcycle, and also had a tendency when riding the Kawasaki to turn the front wheel during low speed cornering, which reduces the motorcycle lean angle required for a given maneuver. The expert riders had similar tendencies, but to a lesser degree. Additional research could, perhaps, develop and test a model that differentiates between and accounts for differences in the lean angle of the rider and the motorcycle.

Two additional areas deserve comment based on the results in Figures 10, 11, 12. First, the Kawasaki motorcycle had a geometric limit of around 30 degrees of lean. Based on Equation (16), this geometric limit should have allowed the riders to reach a lateral acceleration level of 0.5g. While the expert rider achieved a lateral acceleration of 0.46g on this motorcycle and the novice rider achieved 0.45g, the bulk of the data points for this motorcycle fell below 0.4g. Both riders reported repeatedly scraping components of this motorcycle during the testing.

The Suzuki, on the other hand, had a geometric limit exceeding 40 degrees of lean. Neither rider reported scraping components of the motorcycle during the testing with this motorcycle. The expert rider achieved a lateral acceleration level of 0.75g with this motorcycle,

whereas the novice only achieved 0.56g. In the test results with the Kawasaki, it is difficult to discern any difference in the lean angle behavior of the novice and expert riders. With the Suzuki, the novice rider, for the most part, kept the lean angle below 30 degrees. The expert rider used lean angles with this motorcycle that approached and exceed 40 degrees. These results indicate that the observed limits for lean angle on the Kawasaki are being driven by the geometric limit defined by component of this motorcycle contacting the road surface. Initially, it was thought that a psychological limit could be driving the results for the Suzuki. However, in discussing the Suzuki tests with the expert rider, he indicated that the limiting factor for his lean angles on this motorcycle were due to the geometry of the rear tire, not to a psychological limit.

## Conclusions

- 1. The actual motorcycle lean angle employed by the riders in our testing was nearly always greater than the lean angle predicted by Equation (1).
- 2. This means that if <u>Equation (1)</u> were used to calculate the geometry-limited speed of a motorcycle for a particular curve, it would likely overestimate that limit.

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# **APPENDIX**

# **Appendix A - Motorcycle and Rider Information**

Testing Date	4/28/2014	7/7/2014
Motorcycle Make / Model	2012 Suzuki DR650SE Enduro	2007 Kawasaki VN900-D
Front Tire Make / Model	Bridgestone Trail Wing - 41	Dunlop D404
Front Tire Size	90/90-21	130/90-16
Rear Tire Make / Model	Bridgestone Trail Wing - 42	Kenda Cruiser S/T
Rear Tire Size	120/90-17	170/80-15
Motorcycle Weight (lb)	366	665
Expert Rider Weight (lb)	180	183
Novice Rider Weight (Ib)	162	162

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