

## Tire Mark Striations: Sensitivity and Uncertainty Analysis

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### ABSTRACT

Previous work demonstrated that the orientation of tire mark striations can be used to infer the braking actions of the driver [1]. An equation that related tire mark striation angle to longitudinal tire slip, the mathematical definition of braking, was presented. This equation can be used to quantify the driver's braking input based on the physical evidence. Braking input levels will affect the speed of a yawing vehicle and quantifying the amount of braking can increase the accuracy of a speed analysis. When using this technique in practice, it is helpful to understand the sensitivity and uncertainties of the equation. The sensitivity and uncertainty of the equation are explored and presented in this study. The results help to formulate guidelines for the practical application of the method and expected accuracy under specified conditions. A case study is included that demonstrates the analysis of tire mark striations deposited during a real-world accident.

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### INTRODUCTION

Tire marks often fall into one of two categories: skid marks or yaw marks. Skid marks are typically deposited when a driver applies the brakes aggressively. In response to the braking, the tires can slip and deposit rubber on the roadway. Yaw marks are deposited by tires that have generated a slip angle, or a discrepancy between their heading direction and velocity direction (Figure 1). As the tire slides sideways, rubber is deposited on the roadway.

Tire marks sometimes exhibit striations. In the absence of braking or partial braking, yawing tires are simultaneously rolling and sliding which creates striation marks that run at an angle to the direction of the tire mark. In the absence of braking, these striations are aligned at an angle perpendicular to the tires heading direction. In the case of full braking (lock-up) the striations are aligned with the tire mark, or parallel to the wheel hub velocity direction.<sup>1</sup> Because the striations are affected by both the heading angle of the tire and the amount of braking, these tire marks offer a glimpse into the actions of the driver when the tire marks were deposited. A full review of tire striations in the literature can be found in a forthcoming SAE publication [2]

In previous work, a theoretical model for determining longitudinal tire slip from striation marks was developed [1]. Equation 1 can be used to calculate the longitudinal slip, using the striation angle,  $\theta$ , and the slip angle,  $\alpha$ . The variables in Equation 1 are depicted in Figure 2. Full scale vehicle yaw testing was conducted to validate the equation. It was found that the model offered insight into the braking actions of drivers at the time the tire marks were being deposited.



Figure 1. Striated tire marks deposited by a yawing vehicle.

$$S = \frac{\tan \alpha}{\tan(90 - \theta)} \quad (1)$$

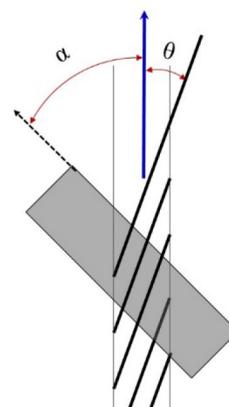


Figure 2. Tire depositing a yaw mark (not to scale).

1. Throughout this paper, the term "full braking" refers to the locking of the wheel, when its rotational velocity goes to zero. This condition would be consistent with full brake application in a vehicle that is not equipped with an antilock brake system (ABS).

## EQUATION SENSITIVITY AND ERROR ANALYSIS

In order to analyze striations in practice, an understanding of the sensitivity of the equations and how error propagates through the analysis would be useful. In this section, the equation for longitudinal slip (Equation 1) will be studied.

### Sensitivity Analysis

Application of the brakes has the effect of changing the angle of the striation marks [1,2,3,4]. Specifically, the marks change from a direction perpendicular to the tire heading with no braking, to a direction parallel to the wheel hub velocity (parallel to the tire mark) with full braking. The change in striation angle with braking is depicted in Figure 3. No braking, partial braking and full braking scenarios are depicted from left to right. In all scenarios the tire is at the same slip angle,  $\alpha$ . The tires are moving from bottom to top on the page, as indicated by the blue arrow. The value  $\kappa$  is also introduced in Figure 3 which represents the angle between perpendicular to the tire and the striation direction. In image on the left of the figure, no braking is occurring. The striations (shown in bold lines) are perpendicular to the tire heading. Mathematically, this case can be described as  $\alpha + \theta = 90$  and  $\kappa = 0$ . Partial braking is depicted in the middle image. The brakes have changed the direction of the striations an angle  $\kappa$  from perpendicular to the tire. In the case of partial braking,  $\alpha + \theta + \kappa = 90$ . On the right, the brakes are applied fully, locking the wheel. With full braking, the striations are parallel to the velocity direction of the wheel hub, which is parallel to the tire mark. Under the full braking condition,  $\theta = 0$  and  $\alpha + \kappa = 90$ .

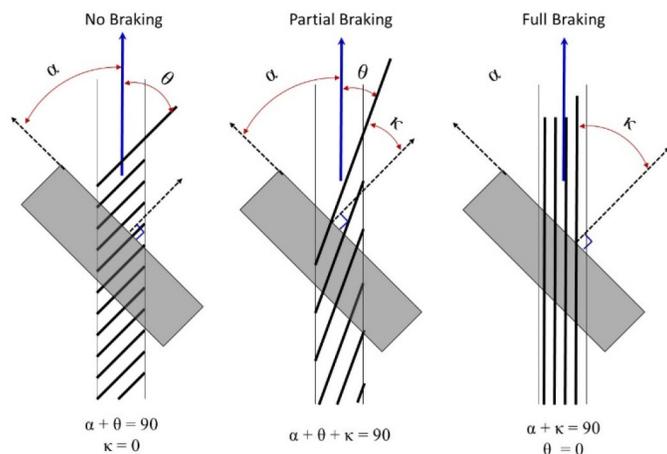


Figure 3. Transition in Striation Direction as a result of Braking (not to scale).

The tires in Figure 3 all have the same slip angle. Now consider the effect of changing the slip angle. Under full braking, the angle  $\kappa$  will be largest when the tire slip angle is small, and vice versa. In other words, when the slip angle is small, there is a relatively large angular difference in  $\kappa$  between no braking and full braking. When the slip angle is large,

there is a relatively small difference in  $\kappa$  between no braking and full braking. In Figure 4, full brake scenarios for slip angles of 10 and 80 degrees are depicted on the left and right, respectively.

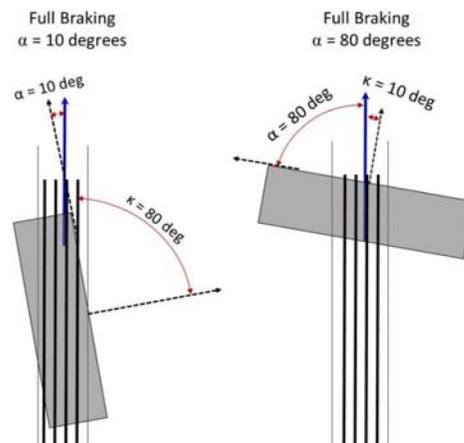


Figure 4. Full Braking, slip angle comparison (not to scale).

In practice, the angle of the striation marks will become more sensitive to braking as the slip angle increases. This sensitivity is illustrated in Figure 5. The longitudinal slip percentage is plotted as a function of the angle  $\kappa$  (Equation 2), for lines of constant slip angles. Maximum deceleration likely occurs at approximately 25 percent longitudinal slip for a typical passenger tire [1]. The build up to maximum braking is shaded in Figure 5. In general, the analysis of striations will be less sensitive to slip and striation angle measurement errors at lower slip angles. At a slip angle of 5 degrees, the striations will change over 70 degrees between no braking and maximum braking and braking will likely be easy to distinguish. At an 85 degree slip angle, less than 2 degrees separate no braking and maximum braking, and changes in striation angle from braking are unlikely to be detected.

$$S = \frac{\tan \alpha}{\tan(90 - \kappa)}$$

(2)

### 3.2. Uncertainty Analysis

An uncertainty analysis of Equation 1 was conducted. The heading angle of the tire, its velocity direction and the striation direction are needed in order to measure  $\alpha$  and  $\theta$ . The velocity direction is used in the measurement of both  $\alpha$  and  $\theta$ . Therefore, the uncertainties in  $\alpha$  and  $\theta$  are correlated. Further, an overestimation of  $\alpha$  will often be accompanied by an underestimation in  $\theta$ , and vice versa. So, the uncertainties in  $\alpha$  and  $\theta$  have negative covariance [5]. In order to account for this, the slip function can be rewritten in a fixed coordinate system. On the left in Figure 6 are the angles  $\alpha$  and  $\theta$  as defined previously. On the right, those two angles are defined system by three measurements,  $A$ ,  $B$ , and  $C$ , from a fixed coordinate.

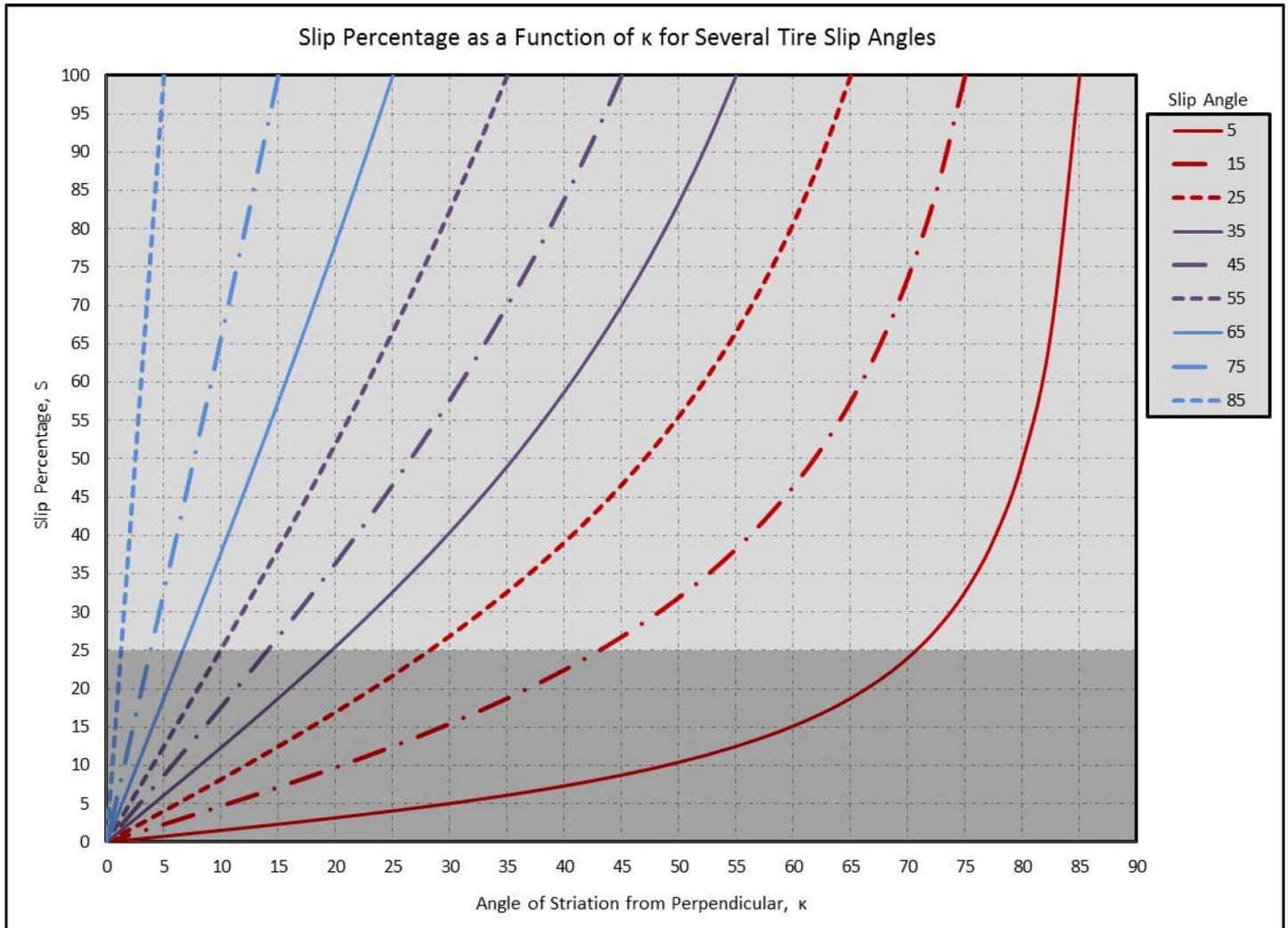


Figure 5. Slip Percentage as a Function of  $\kappa$  for Several Tire Slip Angles.

$$S = \frac{\tan(B - A)}{\tan(C - A)} \tag{3}$$

By rewriting the striation function with the three independent quantities,  $A$ ,  $B$ , and  $C$ , an uncertainty analysis can be conducted that accounts for the negative covariance between the angles  $\alpha$  and  $\theta$ . The uncertainties of three angles,  $\delta A$ ,  $\delta B$ , and  $\delta C$  are considered [5].

$$\delta S = \sqrt{\left(\frac{\partial S}{\partial A} \delta A\right)^2 + \left(\frac{\partial S}{\partial B} \delta B\right)^2 + \left(\frac{\partial S}{\partial C} \delta C\right)^2} \tag{4}$$

The partial derivatives in Equation 4 are defines as follows:

$$\frac{\partial S}{\partial A} = \frac{\sec^2(C - A) \tan(B - A) - \sec^2(B - A) \tan(C - A)}{\tan^2(C - A)} \tag{5}$$

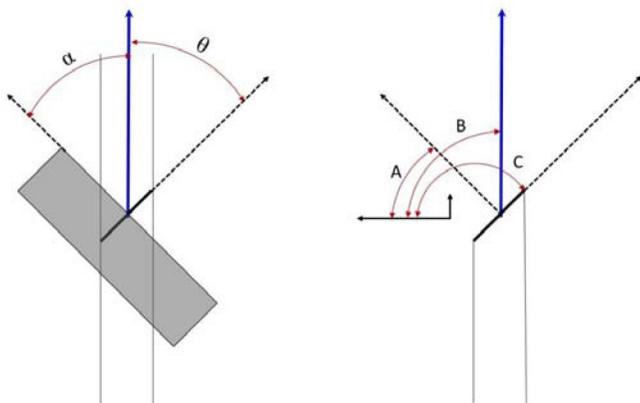


Figure 6. The variables of Equations 1 and 3 (not to scale).

The angles  $\alpha$  and  $\theta$  can then be redefined in terms of  $A$ ,  $B$ ,  $C$ , and the slip function can be rewritten as Equation 3.

$$\frac{\partial S}{\partial B} = \frac{\sec^2(B - A)}{\tan(C - A)} \quad (6)$$

$$\frac{\partial S}{\partial C} = -\frac{\tan(B - A) \sec^2(C - A)}{\tan^2(C - A)} \quad (7)$$

The uncertainty of the slip function was calculated for a variety of slip angle and slip percentage combinations. For the purpose of discussion, the following uncertainties in A, B and C were selected which yield uncertainties in  $\alpha$  and  $\theta$  of +/- 2 degrees. This may be similar to the measurement uncertainties in an actual analysis, but are only chosen for the purpose of observing trends. In practice the uncertainties should be determined on a case by case basis according to the physical evidence. This process will be discussed in the case study presented at the end of the paper.

$\delta A = \pm 1.5^\circ$  (Uncertainty in tire heading)

$\delta B = \pm 0.5^\circ$  (Uncertainty in velocity direction)

$\delta C = \pm 1.5^\circ$  (Uncertainty in striation direction)

Figures 7 through 15 graphically depict slip percentage versus the angle of the striation from perpendicular ( $\kappa$ ) for lines of constant slip angles. Each line in Figure 5 is plotted on its own graph, including the error bars. As in Figure 5, the slip range of 0 to 25% has been highlighted. At a longitudinal slip of 25%, the longitudinal force on the tire is maximized. [6]. The uncertainties analyzed here may be representative of the uncertainties present in a real world analysis. However, the uncertainties can be determined on a case by case basis depending on the characteristics of the tire marks and quality of the documentation of the evidence.

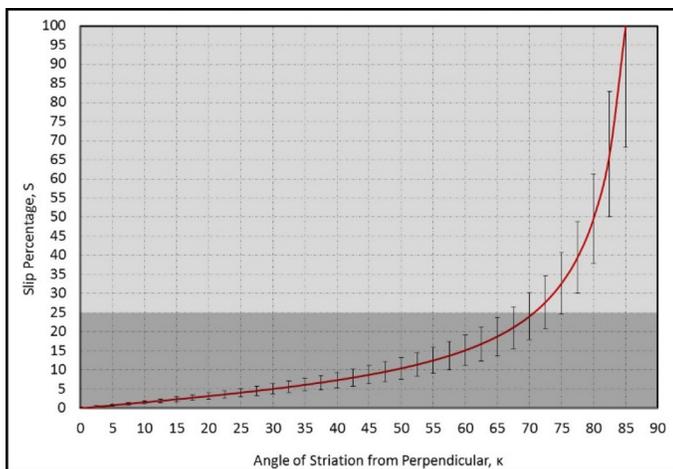


Figure 7. Slip percentage and error rate for 5 degree slip angle.

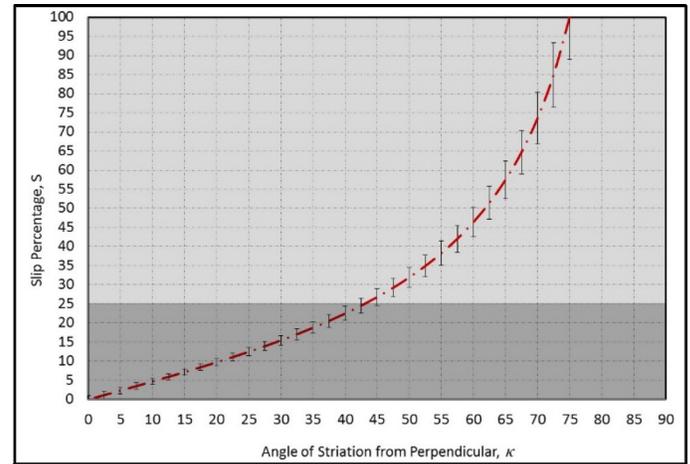


Figure 8. Slip percentage and error rate for 15 degree slip angle.

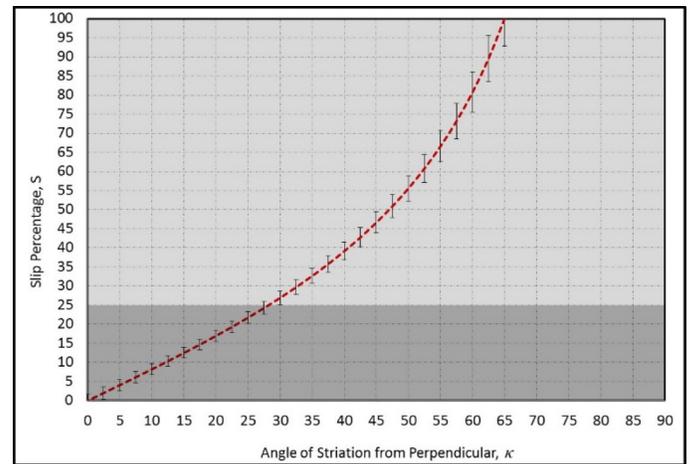


Figure 9. Slip percentage and error rate for 25 degree slip angle.

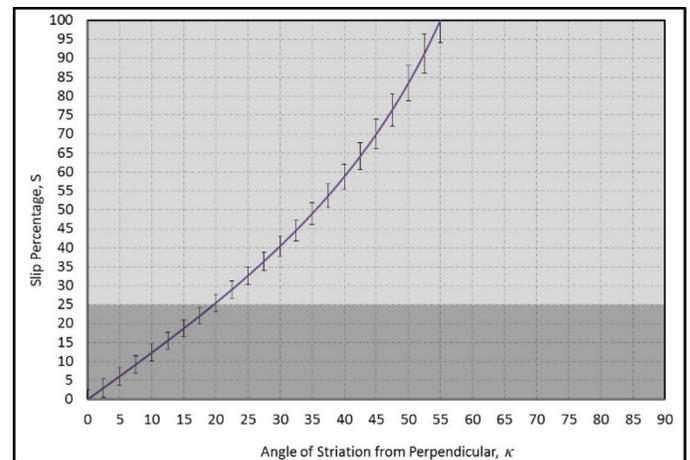


Figure 10. Slip percentage and error rate for 35 degree slip angle.

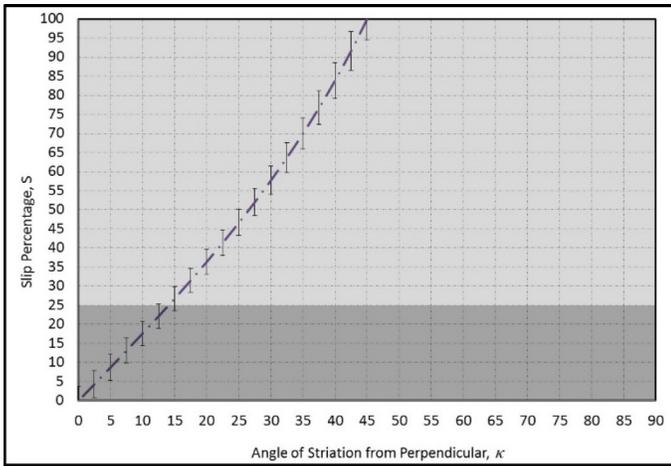


Figure 11. Slip percentage and error rate for 45 degree slip angle.

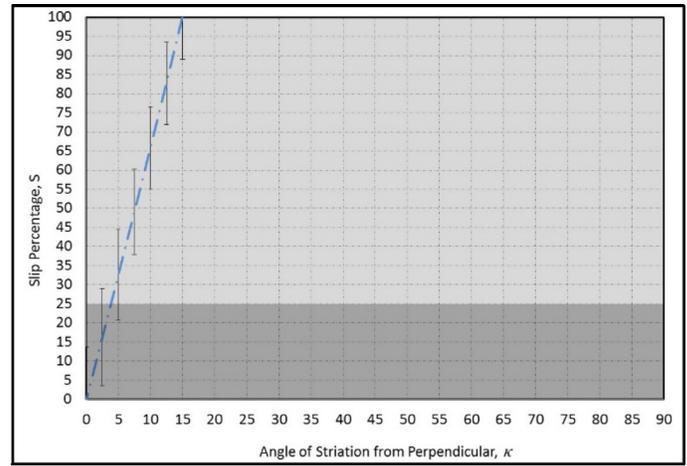


Figure 14. Slip percentage and error rate for 75 degree slip angle.

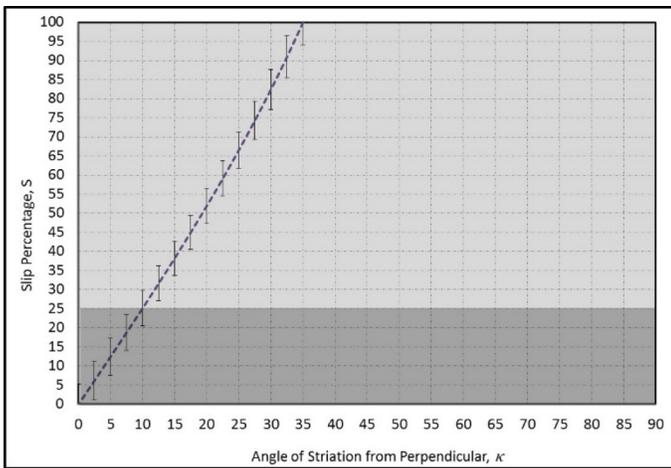


Figure 12. Slip percentage and error rate for 55 degree slip angle.

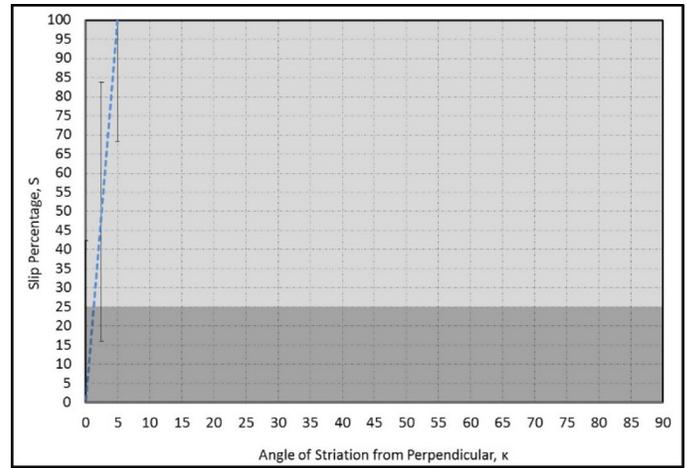


Figure 15. Slip percentage and error rate for 85 degree slip angle.

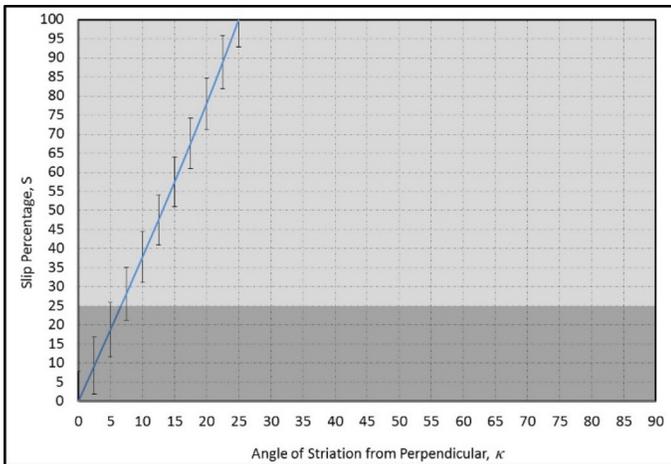


Figure 13. Slip percentage and error rate for 65 degree slip angle.

There are several observations to be made from the plots in [Figures 7](#) through [15](#):

1. Different combinations of slip angle and braking will result in different levels of uncertainty in calculating slip percentage.
2. If striations are perpendicular or nearly perpendicular to the tire ( $\kappa \sim 0$ ), uncertainty in slip percentage increases as slip angle increases.
3. As the brakes are applied and longitudinal slip is generated, uncertainty in slip increased at lower slip angles and decreased at higher slip angles. The transition occurred near a slip angle of 55 degrees.
4. In general, the uncertainty in slip was greatest near wheel lock up (near 100% slip). However, maximum braking is achieved at a slip angle of approximately 25%. Between 25% and 75% slip, there is minimal change in how the vehicle is actually decelerating [[6](#)]. Therefore, greater uncertainty at higher slip percentages is of little consequence to a speed analysis calculation.

5. Uncertainty in the slip calculation was relatively large at the maximum slip angle studied here ( $\alpha = 85$  degrees). It is unlikely that striation analysis of braking would yield dependable results for slip percentage at this slip angle. Calculations of slip percentage from striations at high slip angles may yield non-meaningful results due to the uncertainties in measurement. However, at this slip angle, the vehicle is nearly sliding sideways, and application of the brakes does not contribute to a meaningful reduction in speed. Striation analysis is most sensitive at higher slip angles but, fortunately, a speed analysis is least sensitive to changes in slip at high slip angles.

The previous plots assume a measurement uncertainty of  $\pm 2$  degrees for both slip angle and striation angle. Therefore, some of these observations may not apply to all cases. In practice, the uncertainties in slip can be used to understand a range of potential braking inputs to be used in a speed analysis.

#### 4. CASE STUDY

A real world accident was analyzed to demonstrate the application of striation analysis. In this accident, an SUV departed the left side of the road in a counterclockwise yaw. Upon leaving the road, the vehicle rolled over multiple times before coming to rest on its roof in the center median. The vehicle deposited striated tire marks while it was yawing on the roadway. The initial speed of the vehicle was in question. [Figure 16](#) depicts the tire marks on the roadway and vehicle at rest in the median. The striations in the tire marks are visible, most notably, where the tire passed over the yellow lane line in the lower right corner of the figure.

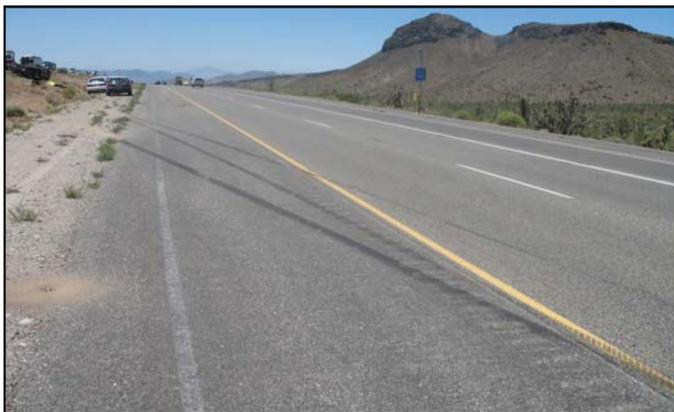


Figure 16. Tire marks and rest position of the vehicle in the case study.

Prior to analyzing the tire marks, a digital accident scene diagram and vehicle model were created. The specific details of creating these models is beyond the scope of this publication. However, in general, this process typically involves these steps:

1. The accident scene is inspected and roadway geometry is mapped.
2. Any physical evidence present at the time of the inspection is mapped.
3. The survey is used to create an accident scene diagram.

4. The striated tire marks are located on the diagram using photogrammetry. Photogrammetry is the process of acquiring measurements from photographs. Through photogrammetry, the photographs are rectified so that they can be viewed from an orthographic top down view and be aligned with the physical evidence.
5. A vehicle model is created based on vehicle specifications.
6. The vehicle model is positioned on the tire marks in the diagram.
7. The striation analysis is then conducted.

[Figure 17](#) depicts the motion of the vehicle as defined by the tire mark evidence. The rest position of the vehicle is also shown. [Figure 18](#) depicts a closer view of the vehicle as it passed across the yellow line. [Figure 18](#) also includes the rectified photograph aligned with the accident scene diagram. [Figure 19](#) focuses on the rear left tire at the time it deposited a striated tire mark on the yellow line.

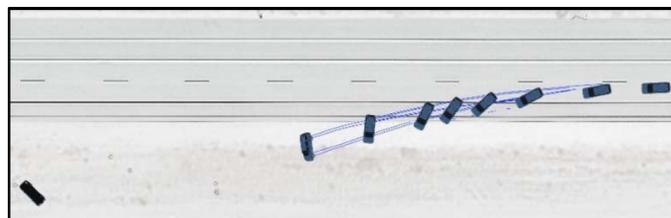


Figure 17. Motion of the vehicle.

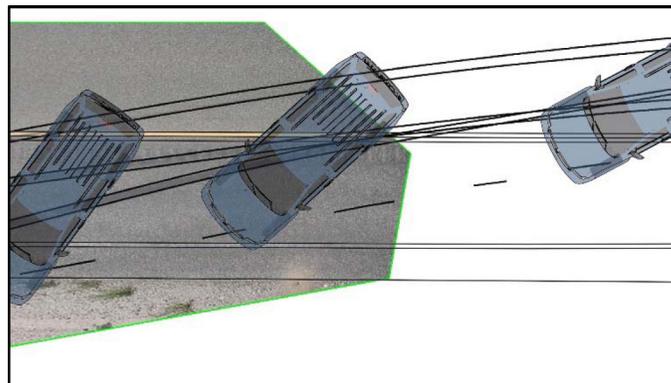


Figure 18. Position of vehicle as its left rear tire crossed the fog line.

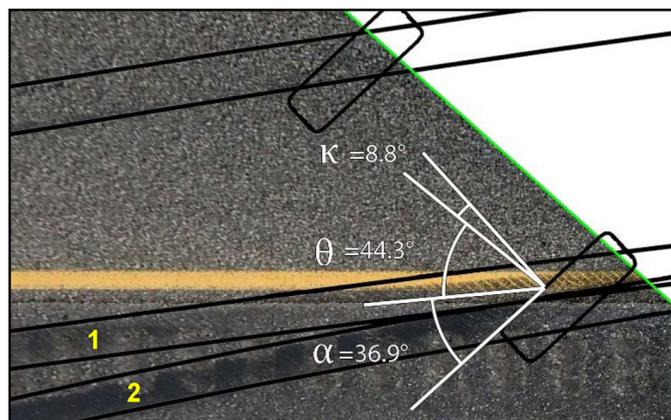


Figure 19. Left rear tire on the striated tire mark along with the measurements of  $\alpha$  and  $\theta$ .

In the absence of braking, the striation marks are aligned perpendicular to the tire heading. Therefore, steering the tires will change the orientation of the striation marks. When assessing braking levels, the rear tires and rear tire marks should be chosen for analysis since the rear tires cannot be steered with the steering wheel. The tire mark deposited by the rear left tire (tire mark 1) will be the focus of this case study.

Once the vehicle model is aligned with the tire mark evidence, the angles  $\alpha(36.9^\circ)$  and  $\theta(44.3^\circ)$  can be measured. Also the uncertainties of the tire heading, velocity direction and striation direction can be approximated. Judgment was used in determining the uncertainties in the measurements. The uncertainties were estimated by observing what range of angles were visually consistent with the physical evidence. A 4 degree range of tire headings still positioned the vehicle on the tire mark evidence, a 2 degree range fit the velocity direction, or tire mark tangent, and a 1 degree range fit with the striation direction. The uncertainties were approximated as the following:

$$\delta A = \pm 2^\circ \text{ (Uncertainty in tire heading)}$$

$$\delta B = \pm 1^\circ \text{ (Uncertainty in velocity direction)}$$

$$\delta C = \pm 0.5^\circ \text{ (Uncertainty in striation direction)}$$

In this case, the longitudinal slip was determined to be  $12 \pm 2$  percent meaning the driver was applying the brakes when these marks were deposited. In analyzing the speed of the vehicle the braking can then be quantified in the form of longitudinal slip. The computed longitudinal slip values can then be used in a speed analysis. [6]

## CONCLUSIONS

The ability to quantify the uncertainty in the analysis of striations has value for the Accident Reconstructionist. As discussed, the magnitude of uncertainty in the slip function depends on the specific conditions when the tire mark was deposited. In the case study, it was shown how the braking actions of the driver can be quantified from the tire marks. Specifically, the result of  $12 \pm 2$  percent slip in this case study indicates that the driver was braking as the vehicle departed the roadway. Traditionally, tire marks have been used to analyze the position of the vehicle and calculate vehicle speed. The striations in the tire marks are an additional piece of evidence that can aid the analyst in refining their speed calculations.

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